# Advanced Higher Physics

# Past Paper Questions

# 2.3 Simple Harmonic Motion


Xkcd.com

**8.** A student is investigating simple harmonic motion. An oscillating mass on a spring, and a motion sensor connected to a computer, are used in the investigation. This is shown in Figure 8A.

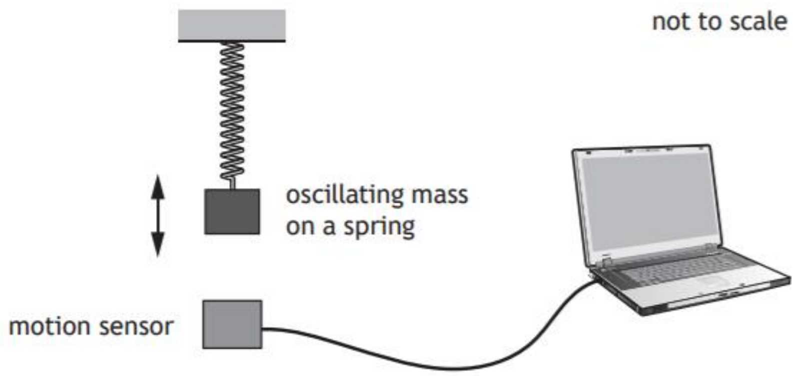



Figure 8A

The student raises the mass from its rest position and then releases it. The computer starts recording data when the mass is released.

(a) The student plans to model the displacement $y$ of the mass from its rest position, using the expression

$$y = A\sin\omega t$$

where the symbols have their usual meaning.

Explain why the student is incorrect. **1**

(b) (i) The unbalanced force acting on the mass is given by the expression

$$F = -m\omega^2 y$$

Hooke's Law is given by the expression

$$F = -ky$$

where $k$ is the spring constant.

By comparing these expressions, show that the frequency of the oscillation can be described by the relationship

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

**2**

(ii) The student measures the mass to be 0·50 kg and the period of oscillation to be 0·80 s.

Determine a value for the spring constant $k$. **3**

**2017**

(iii) The student plans to repeat the experiment using the same mass and a second spring, which has a spring constant twice the value of the original.

Determine the expected period of oscillation of the mass. **2**

(c) The student obtains graphs showing the variation of displacement with time, velocity with time and acceleration with time.

The student forgets to label the y-axis for each graph.

Complete the labelling of the y-axis of each graph in Figure 8B. **2**

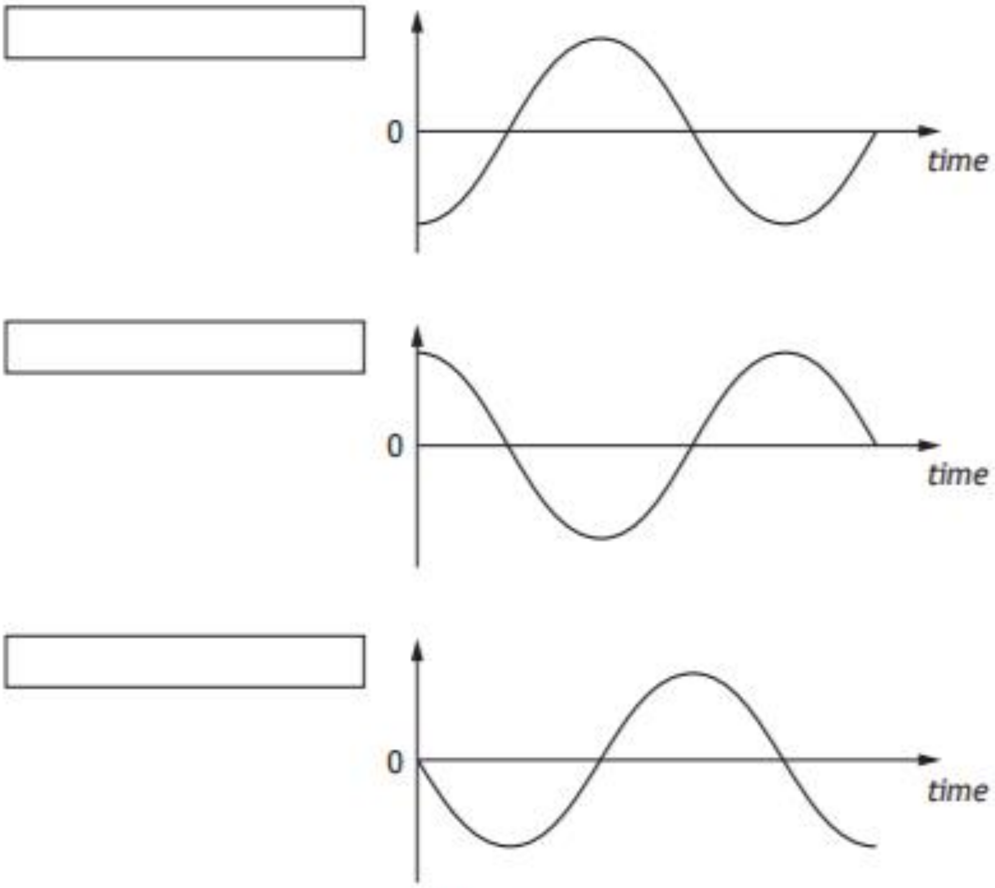



Figure 8B

**10.** (a) (i) State what is meant by *simple harmonic motion*. **1**

(ii) The displacement of an oscillating object can be described by the expression

$$y = A\cos\omega t$$

where the symbols have their usual meaning.

Show that this expression is a solution to the equation

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

**2**

(b) A mass of 1·5 kg is suspended from a spring of negligible mass as shown in Figure 10. The mass is displaced downwards 0·040 m from its equilibrium position.

The mass is then released from this position and begins to oscillate. The mass completes ten oscillations in a time of 12 s.

Frictional forces can be considered to be negligible.

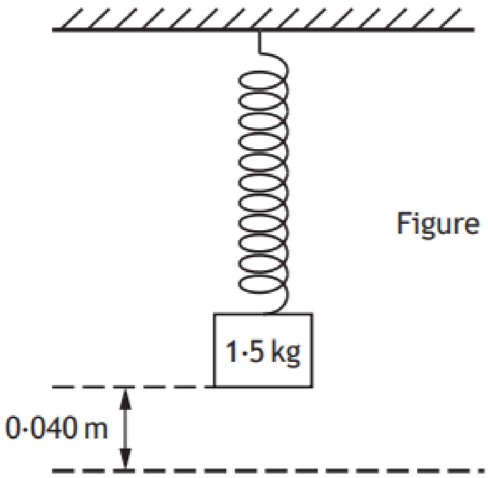



Figure 10

(i) Show that the angular frequency ω of the mass is 5·2 rad s$^{-1}$. **3**

(ii) Calculate the maximum velocity of the mass. **3**

(iii) Determine the potential energy stored in the spring when the mass is at its maximum displacement. **3**

(c) The system is now modified so that a damping force acts on the oscillating mass.

(i) Describe how this modification may be achieved. **1**

(ii) Using the axes below sketch a graph showing, for the modified system, how the displacement of the mass varies with time after release.

Numerical values are **not** required on the axes.


**Specimen Paper**

MARKS

**5.** Figure 5A shows a snowboarder in a half pipe. The snowboarder is moving between positions P and Q. The total mass of snowboarder and board is 85 kg.

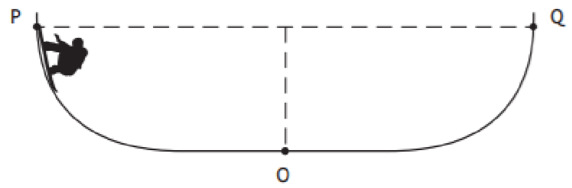
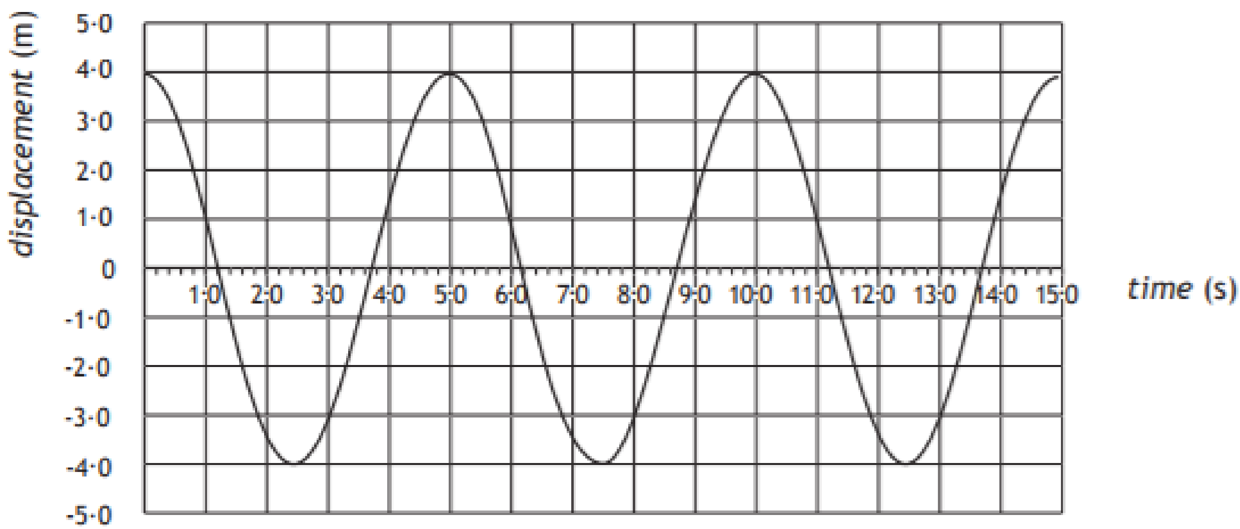



Figure 5A

A student attempts to model the motion of the snowboarder as simple harmonic motion (SHM).

The student uses measurements of amplitude and period to produce the displacement-time graph shown in Figure 5B.



(a) (i) State what is meant by the term *simple harmonic motion*. 1

(ii) Determine the angular frequency of the motion. 4

(iii) Calculate the maximum acceleration experienced by the snowboarder on the halfpipe. 3

(iv) Sketch a velocity-time graph for one period of this motion.

Numerical values are required on both axes. 3

(v) Calculate the maximum potential energy of the snowboarder. 3

(b) Detailed video analysis shows that the snowboarder's motion is not fully described by the SHM model.

Using your knowledge of physics, comment on possible reasons for this discrepancy. 3

**Revised 2015**

*Marks*

6. (*a*) (i) State what is meant by *simple harmonic motion*. 1

(ii) The displacement of an oscillating mass can be described by the expression

$$y = A\sin\omega t$$

where the symbols have their usual meanings.

Show that this mass exhibits simple harmonic motion (SHM). 2

(iii) The displacement of an object exhibiting SHM can also be written as

$$y = A\cos\omega t$$

Identify the initial condition for which this equation would be used. 1

(*b*) A mass attached to a spring is displaced from its equilibrium position and allowed to oscillate vertically. A motion sensor, connected to a computer, is placed below the mass as shown in Figure 6A.


Figure 6A

Figure 6B shows the graph of the displacement from equilibrium position against time for the mass.

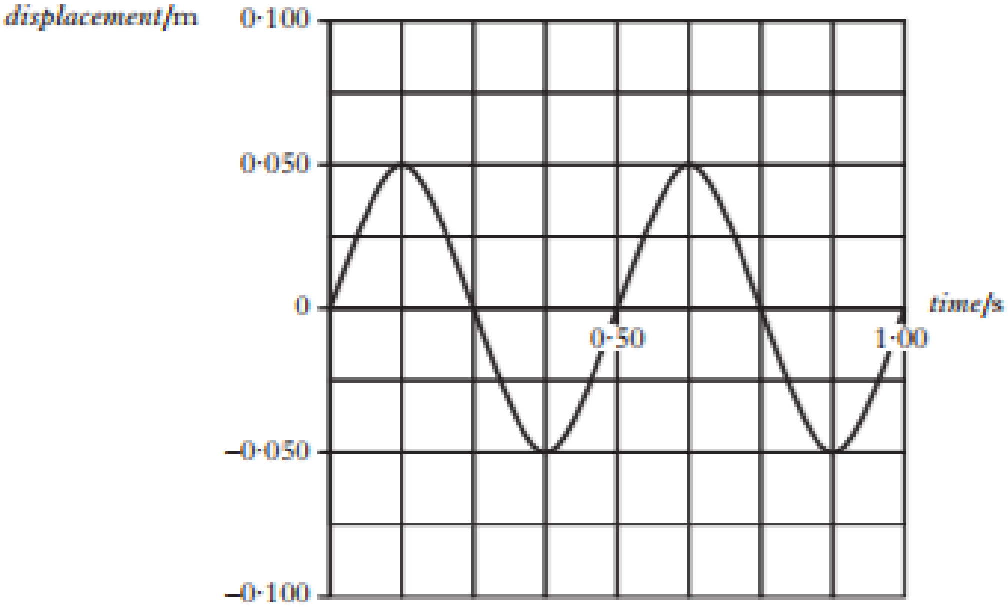



Figure 6B

(i) Using data from the graph, determine the velocity of the mass at 0·50 s. **3**

(ii) Calculate the maximum acceleration of the mass. **2**

(*c*) The system is modified by attaching a rigid card of negligible mass as shown in Figure 6C.

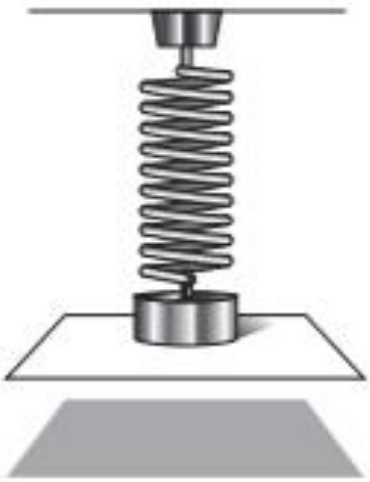


Figure 6C

The mass is displaced from its equilibrium position and allowed to oscillate vertically.

Sketch a displacement time graph of this motion. **1**

**(10)**

**5.** Figure 5A shows a snowboarder in a half pipe. The snowboarder is moving between positions P and Q. The total mass of snowboarder and board is 85 kg.

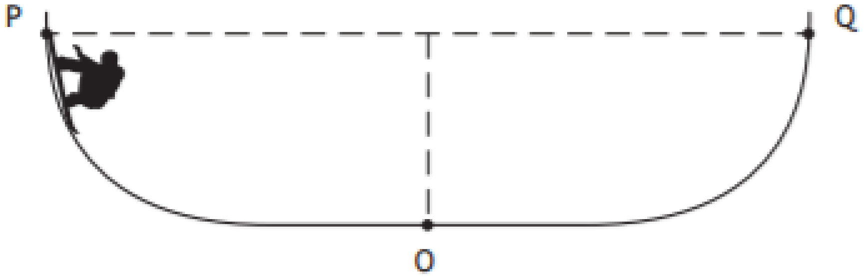



Figure 5A

A student attempts to model the motion of the snowboarder as simple harmonic motion (SHM).

The student uses measurements of amplitude and period to produce the displacement-time graph shown in Figure 5B.

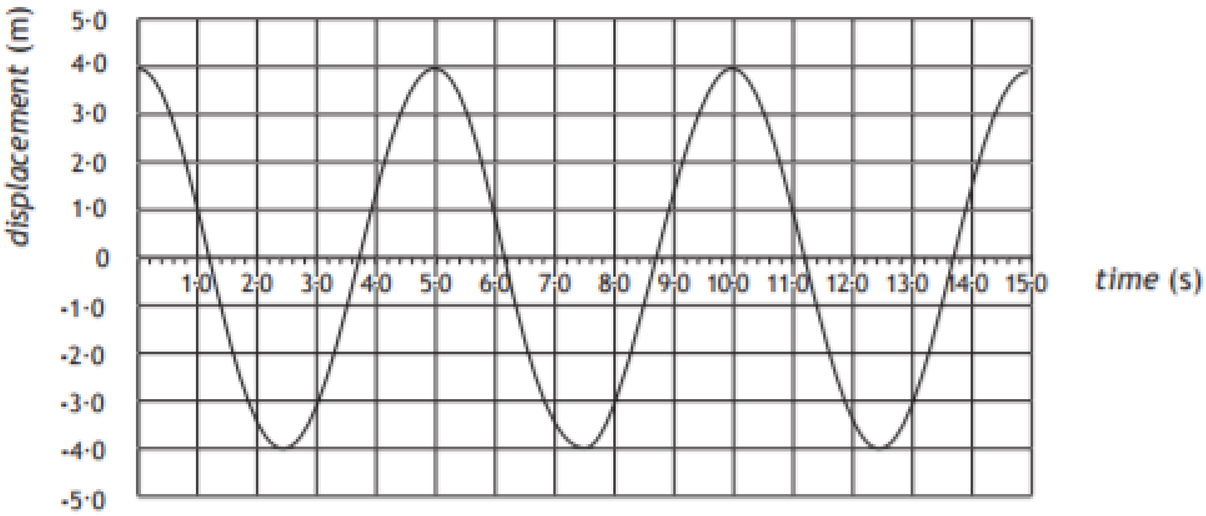



Figure 5B

(a) (i) State what is meant by the term *simple harmonic motion*. **1**

(ii) Determine the angular frequency of the motion. **4**

(iii) Calculate the maximum acceleration experienced by the snowboarder on the halfpipe. **3**

(iv) Sketch a velocity-time graph for one period of this motion.

Numerical values are required on both axes. **3**

(v) Calculate the maximum potential energy of the snowboarder. **3**

(b) Detailed video analysis shows that the snowboarder's motion is not fully described by the SHM model.

Using your knowledge of physics, comment on possible reasons for this discrepancy. **3**

**8.** Car engines use the ignition of fuel to release energy which moves the pistons up and down, causing the crankshaft to rotate.

The vertical motion of the piston approximates to simple harmonic motion.

Figure 8 shows different positions of a piston in a car engine.

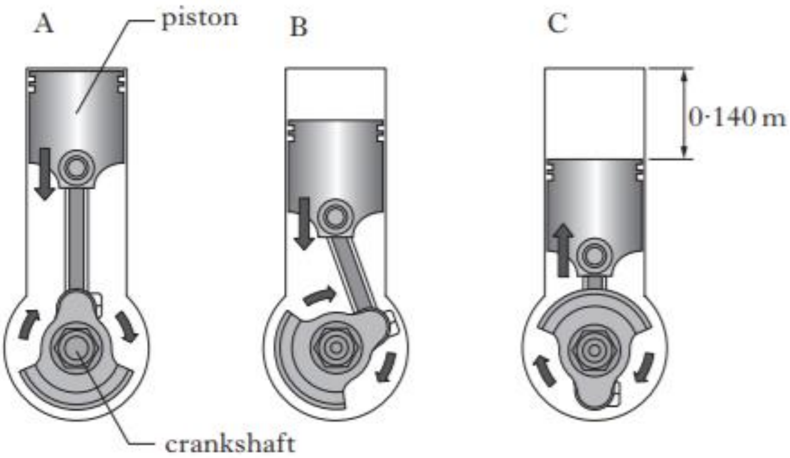



Figure 8

(*a*) Define *simple harmonic motion*. **1**

(*b*) Determine the amplitude of the motion. **1**

(*c*) In this engine the crankshaft rotates at 1500 revolutions per minute and the piston has a total mass of 1·40 kg.

(i) Calculate the maximum acceleration of the piston. **3**

(ii) Calculate the maximum kinetic energy of the piston. **2**

**(7)**

*Marks*

**14.** A group of students were evaluating an experiment to investigate the relationship between the mass on a spring and its period of oscillation. Figure 14 shows some of the apparatus used.

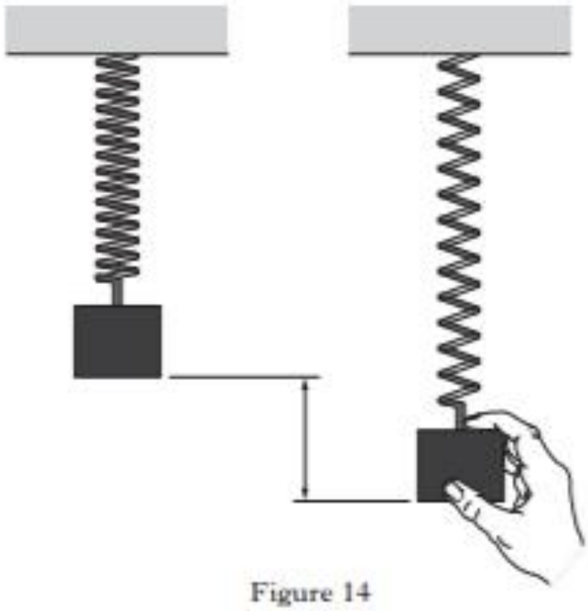


Figure 14

Student A stated "*I think we should use a balance that reads to 0·001 g instead of 0·1 g. This will give us a more accurate answer.*"

Student B stated "*I think we should repeat the time measurement and calculate a mean value.*"

Student C stated "*I think we should time the pendulum for 10 oscillations and divide this value by 10 to get the time for one complete oscillation. This will give us a more precise answer.*"

Student D stated "*I think it would be good to check the mass on another balance.*"

Using your knowledge of experimental analysis, comment on these statements. **(3)**

**7.** A "saucer" swing consists of a bowl shaped seat of mass 1·2 kg suspended by four ropes of negligible mass as shown in Figure 7A.

Figure 7A

When the empty seat is pulled back slightly from its rest position and released its motion approximates to simple harmonic motion.

(*a*) Define the term *simple harmonic motion*. **1**

(*b*) The acceleration-time graph for the seat with no energy loss is shown in Figure 7B.

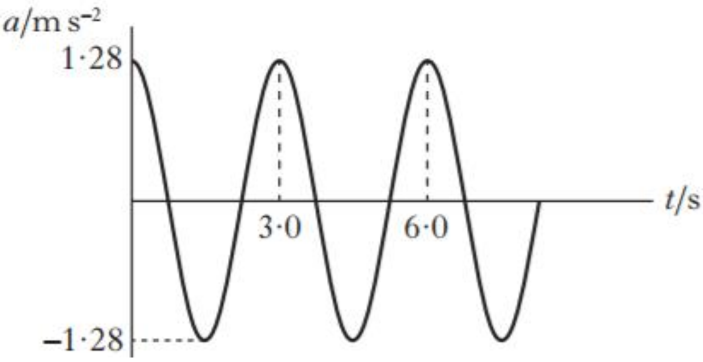



Figure 7B

(i) Show that the amplitude of the motion is 0·29 m. **3**

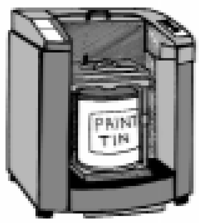
(ii) Calculate the velocity of the seat when its displacement is 0·10 m. **2**

(*c*) Calculate the displacement of the seat when the kinetic energy and potential energy are equal. **3**

**(9)**

5. A motorised mixer in a DIY store is used to mix different coloured paints.

Paints are placed in a tin and the tin is clamped to the base as shown in Figure 5A.

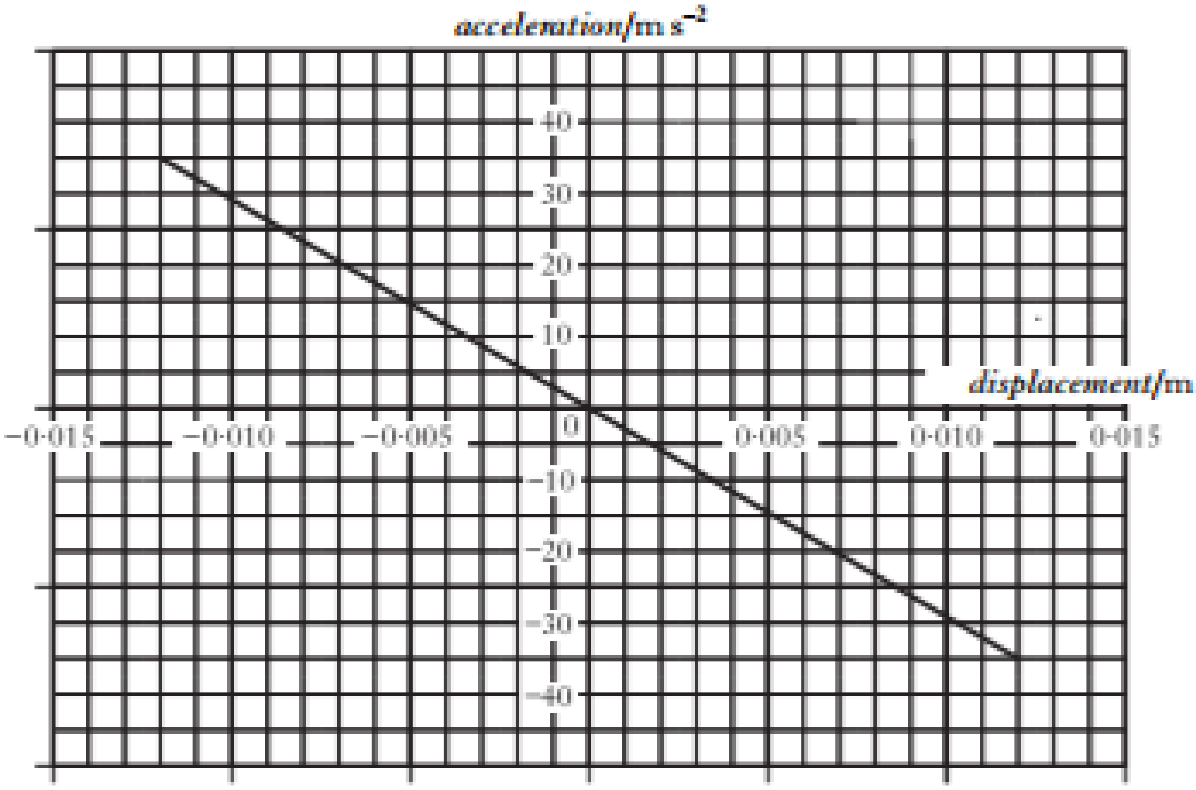



The oscillation of the tin in the vertical plane closely approximates to simple harmonic motion.

The amplitude of the oscillation is 0·012 m.

The mass of the tin of paint is 1·4 kg.

Figure 5B shows the graph of the acceleration against displacement for the tin of paint.




(*a*) Show that the angular frequency $\omega$ of the oscillation is 54 rad $s^{-1}$. **1**

(*b*) Write an expression for the displacement $y$ of the tin as a function of time.
Include appropriate numerical values. **2**

(*c*) Derive an expression for the velocity $v$ of the tin as a function of time.
Numerical values should again be included. **2**

(*d*) Calculate the maximum kinetic energy of the tin of paint as it oscillates. **2**

**(7)**

Marks

3. An electric toothbrush has a circular brush head, diameter 12 mm, as shown in Figure 3A.

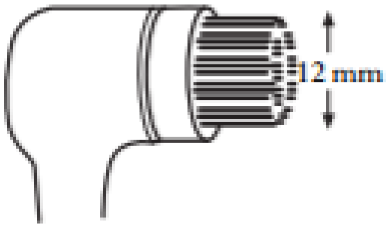



Figure 3A

The toothbrush has two settings.

On setting 1, the brush head vibrates with simple harmonic motion. On this setting, the head vibrates with a frequency of 33 Hz and moves a maximum distance of 4·2 mm as shown in Figure 3B.

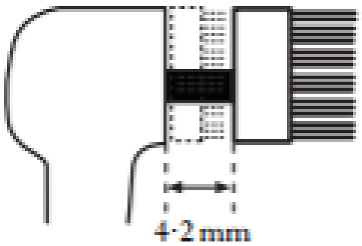



Figure 3B

(a) (i) Write an expression for the displacement, in metres, of the brush head. **2**

(ii) Calculate the maximum speed of the brush head. **2**

**3.** A simple pendulum consists of a lead ball on the end of a long string as shown in Figure 5.


Figure 5

The ball moves with simple harmonic motion. At time $t$ the displacement $s$ of the ball is given by the expression

$$s = 2 \cdot 0 \times 10^{-2} \cos 4 \cdot 3t$$

where $s$ is in metres and $t$ in seconds.

(*a*) (i) State the definition of *simple harmonic motion*. **1**

(ii) Calculate the period of the pendulum. **2**

(*b*) Calculate the maximum speed of the ball. **2**

(*c*) The mass of the ball is $5 \cdot 0 \times 10^{-2}$ kg and the string has negligible mass.

Calculate the total energy of the pendulum. **2**

(*d*) The period $T$ of a pendulum is given by the expression

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where $L$ is the length of the pendulum.

Calculate the length of this pendulum. **2**

(*e*) In the above case, the assumption has been made that the motion is not subject to *damping*.

State what is meant by *damping*. **1**

**(10)**

Marks

4. (*a*) State what is meant by *simple harmonic motion*. 1

(*b*) The motion of a piston in a car engine closely approximates to simple harmonic motion.

In a typical engine, the top of a piston moves up and down between points A and B, a distance of 0·10 m, as shown in Figure 4.

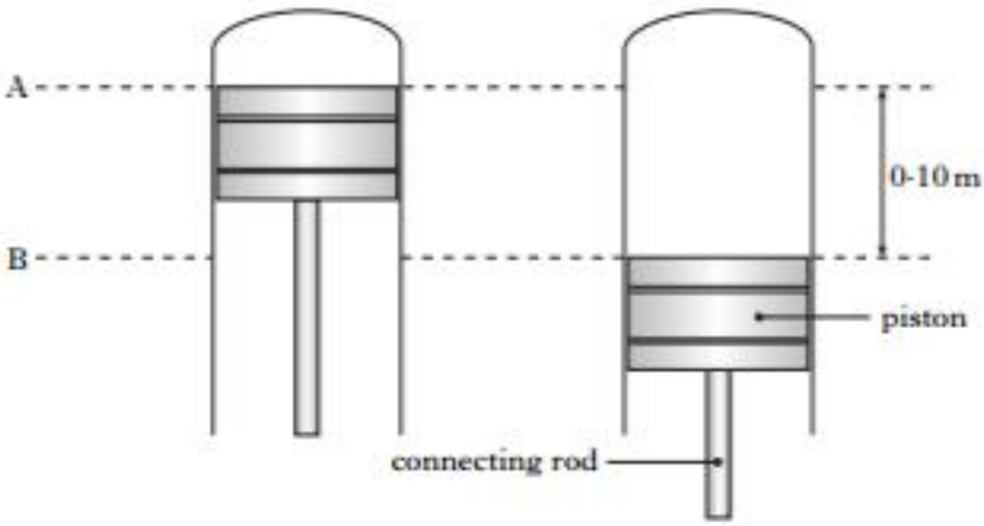



Figure 4

The frequency of the piston's motion is 100 Hz.

Write down an equation which describes how the displacement of the piston from its central position varies with time. Numerical values are required. 2

(*c*) Calculate the maximum acceleration of the piston. 2

(*d*) The mass of the piston is 0·48 kg.

Calculate the maximum force applied to the piston by the connecting rod. 2

(*e*) Calculate the maximum kinetic energy of the piston. 2

(9)

*Marks*

**4.** A test tube contains lead shot. The combined mass of the test tube and the lead shot is 0·250 kg.

The test tube is gently dropped into a container of water and oscillates above and below its equilibrium position with simple harmonic motion as shown in Figure 7.

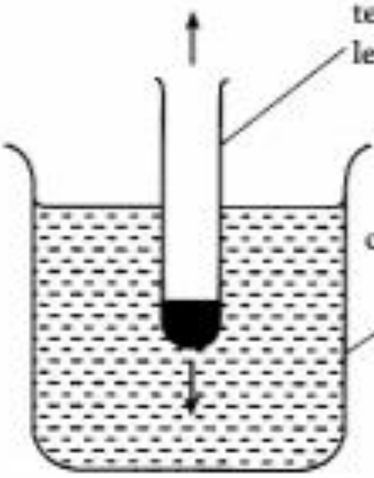



Figure 7

The displacement $y$ of the test tube from its equilibrium position is described by the equation

$$y = 0{\cdot}05 \cos 6t$$

where $y$ is in metres and $t$ is in seconds.

(*a*) Show that the kinetic energy of the test tube, in joules, is given by the equation

$$E_k = 4{\cdot}5\,(2{\cdot}5 \times 10^{-3} - y^2).$$ **2**

(*b*) Calculate the maximum value of the kinetic energy of the test tube. **1**

(*c*) Calculate the potential energy of the test tube when it is 40 mm above its equilibrium position. **2**

**(5)**

Marks

4. The flexible paper cone of a loudspeaker vibrates, producing a sound. The loudspeaker has a small cap at the centre of the cone as shown in Figure 5.

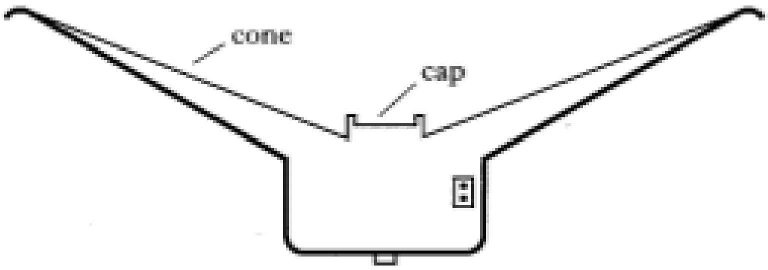



Figure 5

The cone and cap vibrate with simple harmonic motion when the loudspeaker is connected to a signal generator.

(*a*) State what is meant by *simple harmonic motion*. **1**

(*b*) At a particular frequency, the velocity of the cap, in $m\,s^{-1}$, is given by the expression

$$v = 0{\cdot}50 \cos 625t.$$

(i) Calculate the frequency of the sound emitted by the loudspeaker. **2**

(ii) Calculate the amplitude of vibration of the loudspeaker cap. **2**

(*c*) A small polystyrene sphere is placed on the cap of the loudspeaker as shown in Figure 6.

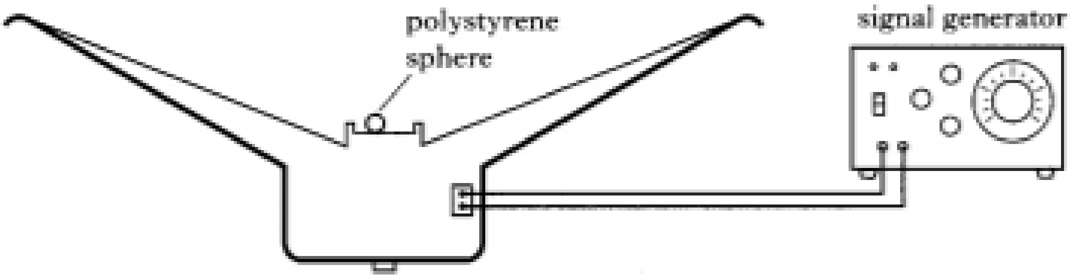



Figure 6

The frequency of the signal generator is slowly increased from zero. At low frequencies the polystyrene sphere stays in contact with the cap. At one particular frequency the sphere just loses contact with the cap. State the maximum acceleration of the cap when this occurs. Justify your answer. **2**

**(7)**

Marks

5. A mass of 0·50 kg is suspended from a spring of negligible mass, as shown in Figure 5(*a*).

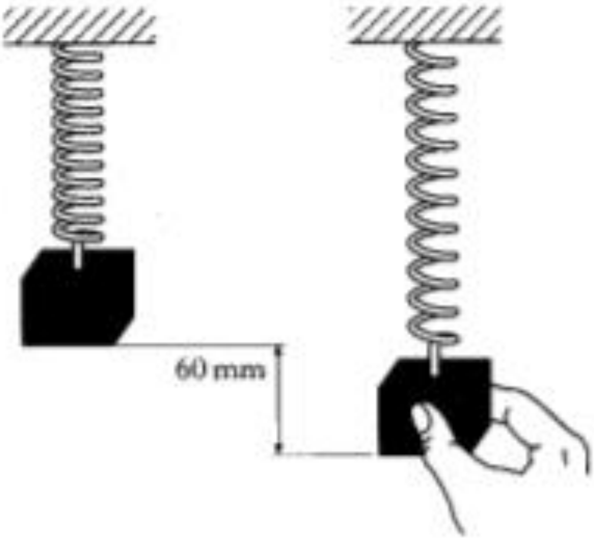



Figure 5(*a*) Figure 5(*b*)

A student pulls the mass down a distance of 60 mm and holds it in the position shown in Figure 5(*b*).

The tension in the spring is now 7·0 N.

(*a*) By considering the vertical forces acting on the mass, calculate the force applied by the student to hold the mass in this position. **1**

(*b*) The student releases the mass, which performs simple harmonic motion.

(i) State the relationship between the unbalanced force acting on the mass and the displacement of the mass.

(ii) Calculate the acceleration of the mass immediately after its release.

(iii) State the initial amplitude of the oscillations. **4**

(*c*) The oscillations of the mass are described by the equation

$$\frac{d^2y}{dt^2} = -\omega^2 y.$$

(i) Name the physical quantity represented by the term $\frac{d^2y}{dt^2}$.

(ii) Calculate the frequency of the oscillations. **3**

**(8)**

Marks

**4.** A mass of 0·40 kg is suspended from a spring as shown in Figure 2. The mass is then displaced vertically and released. Its subsequent motion is recorded using a motion sensor linked to a computer.

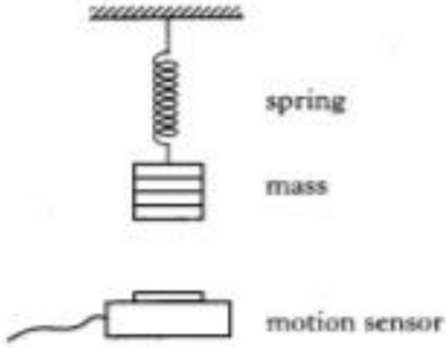



Figure 2

The mass moves with simple harmonic motion. The displacement-time graph of the mass is shown in Figure 3.

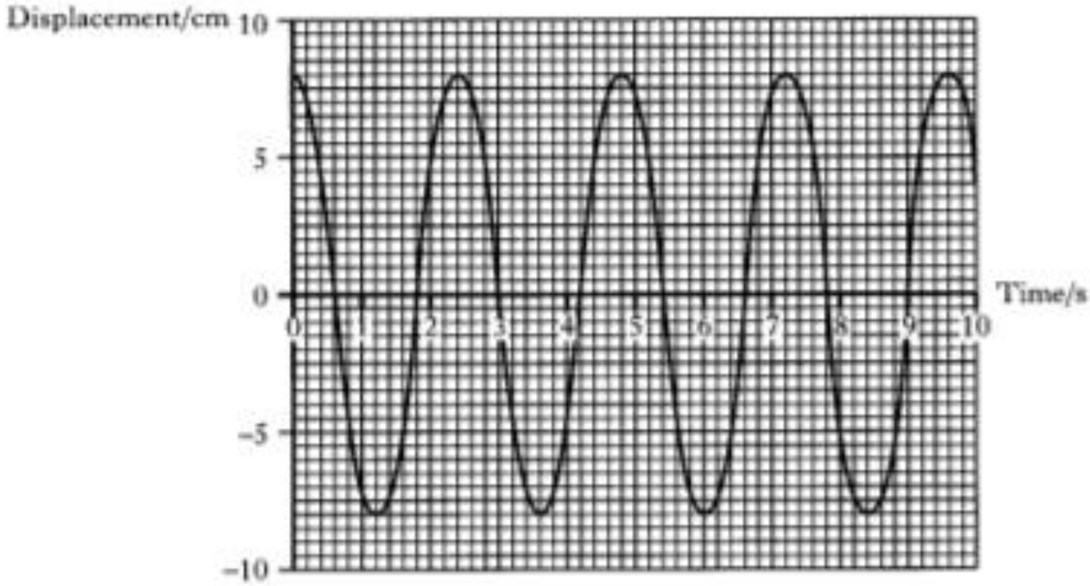



Figure 3

(*a*) Find:

(i) the amplitude of the oscillation;

(ii) the period of the oscillation. **2**

(*b*) Using the values from part (*a*), obtain an expression, in the form $y = A\cos\omega t$, for the vertical displacement $y$ of the mass. **2**

(*c*) (i) Using the solution to part (*b*), derive an expression which gives the relationship between the acceleration $a$ of the mass and time $t$.

(ii) Calculate the maximum kinetic energy of the mass. **5**

**(9)**

4. (*a*) A mass is suspended from a spring and is at rest.

The mass is displaced 20 mm from its rest position, as shown in Figure 7.

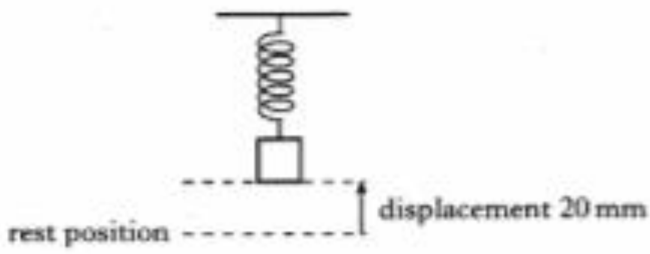



Figure 7

The mass is released.

A graph of the displacement *y* of the mass against time *t* is shown in Figure 8.

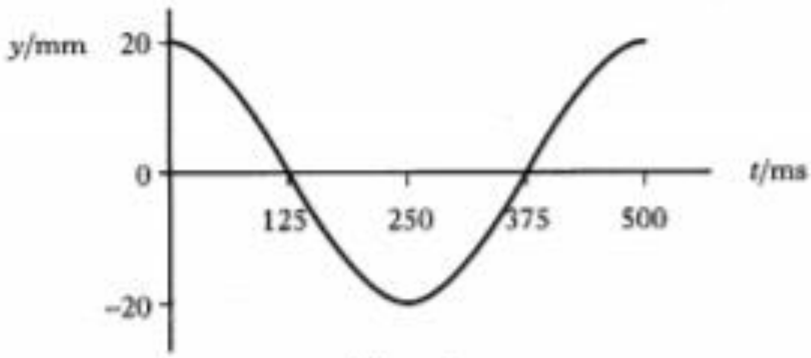



Figure 8

(i) Show, by calculation, that

$$\frac{d^2y}{dt^2} = -158y.$$

(ii) Sketch a graph of the velocity of the mass against time for the first period of the oscillation. Numerical values are required on both axes. **5**

(*b*) An object has a periodic motion and oscillates between A and B as shown in Figure 9.

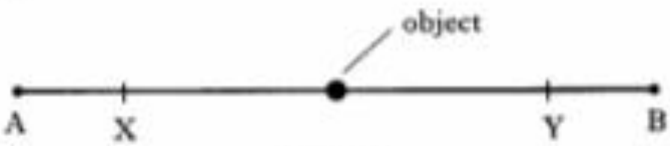



Figure 9

Between points X and Y the object moves with constant speed. Explain **fully** why the motion of the object cannot be described as simple harmonic. **2**

**(7)**